costly design iterations and balanced configurations are neither desirable nor possible.

Acknowledgment

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A Simplified "Real Frequency" Technique Applied to Broad-Band Multistage Microwave Amplifiers

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***Abstract*—A computer-aided design (CAD) procedure, which is a new and simplified "real frequency" technique, is introduced for treating the broad-band matching of an arbitrary load to a complex generator. The method can be applied to the design of interstage equalizers for microwave amplifiers. It utilizes the measured data obtained from the generator and the load networks. Neither an *a priori* choice of an equalizer topology, nor an analytic form of the system transfer function, is assumed. The optimization process of the design procedure is carried out directly in terms of a physically realizable, unit normalized reflection coefficient which describes the equalizer alone.**

Based on the load-generator matching technique, a sequential procedure to design multistage microwave amplifiers is presented. An example is given for a three-stage, FET amplifier proceeding directly from the measured scattering parameters of the FET devices. The example is in three parts and illustrates the sequential method; that is, first a single-stage, then a two-stage, and finally the three-stage system is computed.

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I. Introduction

In the design of broad-band, multistage microwave amplifiers a fundamental problem is to realize lossless interstage equalizers as well as front-end and back-end equalizers so that the transfer of power from source to load is maximized over a prescribed frequency band. In such a case the problem is one of "double matching" power transfer from a complex generator to an arbitrary load (Fig. 1). In this paper, we will first introduce a new computer-aided design procedure, a simplified "real frequency" technique for double-matching problems, then we will extend the technique to the design of broad-band, multistage FET amplifiers.

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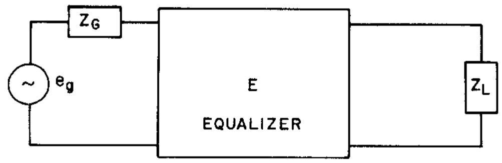


Fig. 1. Double-matching problem.

The double-matching method described here is an alternate version of the "direct computational" technique [3], [9]. The latter in turn is a generalized form of "real frequency single matching" introduced by Carlin [1], [2]. Therefore, it is appropriate to summarize the single-matching real frequency technique briefly. The reader is also referred to [10] for further background material on analytic and real frequency matching methods.

In the real frequency technique for single-matching problems, the resistively terminated equalizer E is described by its positive real (PR) input impedance $Z_q(j\omega) = R_q(\omega) + jX_q(\omega)$. It is apparent that $Z_q(j\omega)$ measured at the load but looking back through the equalizer E towards the resistive generator determines all the transfer properties of the system. Once the impedance $Z_L(j\omega)$ is known either directly from measured data or a circuit model, the transducer power gain $T(\omega)$ written in terms of Z_q and Z_L is determined. In this approach, Z_q is assumed minimum reactance (or equivalently $Y_q = 1/Z_q = G_q + jB_q$ is minimum suseptance). Thus, equalizer reactance $X_q(\omega)$ is determined by a Hilbert transformation once the equalizer resistance $R_q(\omega)$ is found. In other words, the entire problem is formulated as the determination of $R_q(\omega)$ alone to satisfy the gain constraints; e.g., to make $T(\omega)$ as flat and as high as possible over a prescribed band. Further, it is not difficult to treat the nonminimum reactance case. At no stage of the design procedure is it necessary to invent a transfer function which combines both equalizer characteristics and load. Nor is it necessary to assume an equalizer topology. The experimental data for devices to be broadbanded are processed directly. The papers published on this procedure describe simple ways of representing the unknown $R_q(\omega)$, e.g., line segments, to facilitate the optimization procedure.

The simplified real frequency technique described in the present paper has all the merits of the above summarized real frequency technique. However, in double matching, the final result of the new procedure is an optimized, physically realizable unit normalized reflection coefficient $e_{11}(s)$ which describes the equalizer alone. That equalizer is then placed between complex source and complex load. The basis of the improved numerical method is to start directly with $e_{11}(s)$ (rather than a resistance function) as the initialized function which is to be finally computed so as to optimize gain-bandwidth from source to load.

As contrasted with [1], [2], [3], and [9] in the new design procedure, numerical evaluation of the Hilbert transformation between R and X is eliminated. It is the actual reflection factor $e_{11}(s)$ $[s = \sigma + j\omega]$ which is directly computed in rational form. The computational efficiency is thereby simplified and improved. Furthermore, just as in the other versions of the real frequency method, the equalizers which result are generally simpler with superior gain properties as compared to structures obtained by the analytic procedure [4].

II. The Simplified Real Frequency Technique

The basis for the double-matching design procedure to be described below is to deal with both the optimization of system transducer gain between complex loads, as well as the realization of the equalizer, directly in terms of the unit normalized reflection factor of the equalizer alone, $e_{11}(s)$. If $e_{11}(s)$ is appropriately determined then the equalizer E may be synthesized using the Darlington theorem that any rational bounded real (BR) nonunitary real normalized reflection coefficient $e_{11}(s)$ is realizable as a lossless reciprocal two-port E terminated in a pure resistance [8]. For simplicity, E is assumed to be a minimum phase structure with transmission zeros only at $\omega = \infty$, $\omega = 0$. This is a convenient assumption since it assures realization without coupled coils, except possibly for an impedance level transformer. The algorithm to be described is generally applicable to all double-matching problems since it neither involves equalizer element values nor equalizer topology.

First, suppose $e_{11}(s)$ is given as

$$e_{11}(s) \triangleq \frac{h(s)}{g(s)} \frac{h_0 + h_1 s + \cdots + h_n s^n}{g_0 + g_1 s + \cdots + g_n s^n} \tag{1}$$

where n specifies the number of total reactive elements in E. Then, employing the well-known Belevitch representation [5], the real normalized scattering parameters of E are given as

$$e_{11}(s) = \frac{h(s)}{g(s)} \tag{2a}$$

$$e_{21}(s) = e_{12}(s) = \pm \frac{s^k}{g(s)} \tag{2b}$$

$$e_{22}(s) = -(-1)^k \frac{h(-s)}{g(s)} \tag{2c}$$

where $k \geqslant 0$ is an integer and specifies the order of the zero of transmission (i.e., of $e_{12}(s)$) at $s = \infty$.

Since the matching network is lossless, it follows that

$$g(s)g(-s) = h(s)h(-s) + (-1)^k s^{2k}. \tag{3a}$$

In the iterative approach presented below, the coefficients of the numerator polynomial $h(s)$ are chosen as unknowns. In order to construct the scattering parameters of E, it is sufficient to generate the Hurwitz denominator polynomial $g(s)$ from $h(s)$. In the following paragraph it will be shown that once the coefficients of $h(s)$ are initialized at the start of the optimization process and the complexity of E is specified (i.e., n and k are fixed), $g(s)$ can be generated as a Hurwitz polynomial by explicit factorization of (3a).

In choosing the polynomial $h(s)$ and the integer k we cannot allow $h(0) = 0$ and $k \neq 0$ simultaneously since this violates the lossless criterion, i.e., $|e_{11}(j\omega)|^2 + |e_{21}(j\omega)|^2 = 1$.

With this restriction satisfied (3a) on $j\omega$ yields

$$|g(j\omega)|^2 = |h(j\omega)|^2 + \omega^{2k} > 0 \tag{3b}$$

and

$$|g(j\omega)|^2 \geqslant |h(j\omega)|^2. \tag{3c}$$

The condition (3b) guarantees a Hurwitz factorization to obtain $g(s)$ and together with (3c) assures the BR character of $e_{11}(s)$ and the lossless realization of E.

The transducer power gain $T(\omega)$ of the doubly terminated structure (Fig. 1) may now be constructed in terms of the e_{ij} and the given complex terminations using the following algorithm.

Algorithm: Computation of $T(\omega)$ from the given numerator polynomial $h(s)$ of the input reflection coefficient $e_{11}(s)$.

Inputs:

- n Degree of the polynomial $h(s) = h_0 + h_1 + \cdots + h_n s^n$.
- k Degree of the numerator polynomial of $e_{21}(s) = e_{12}(s) = \mp s^k / g(s)$.
- $h_0, h_1 \cdots h_n$ Unknown but initialized real coefficients of $h(s)(h(0) \neq 0$ for $k > 0)$.
- $S_G(j\omega)$ Real normalized reflection coefficient of the source network. ($S_G(j\omega)$ is assumed to be BR and the corresponding $Z_G = (1+S_G)/(1-S_G)$ is positive real (PR)).
- $S_L(j\omega)$ Real normalized reflection coefficient of the load network (S_L is assumed to be BR and the corresponding $Z_L = (1+S_L)/(1-S_L)$ is PR).

Computational Steps:

1) Generate the polynomial $g(s)g(-s) = h(s)h(-s) + s^{2k} = G_0 + G_1 s + G_2 s^2 + \cdots + G_n s^n$ where

$$G_0 = h_0^2$$

$$G_1 = h_1^2 + 2h_2 h_0$$

$$\vdots$$

$$G_i = h_i^2 + 2\left(h_{2i}h_0 + \sum_{j=2}^{i} h_{j-1}h_{2i-j+1}\right)$$

$$\vdots$$

$$G_k = G_i|_{i=k} + 1$$

$$\vdots$$

$$G_n = h_n^2.$$

2) Find the roots of $g(s)g(-s)$.

3) Choose the LHP roots of $g(s)g(-s)$, and form the polynomial $g(s) = g_0 + g_1 s + \cdots + g_n s^n$.

4) Construct the real normalized scattering parameters $e_{ij}(s)$ from $h(s)$ and $g(s)$

$$e_{11}(s) = \frac{h(s)}{g(s)}, \quad e_{21} = e_{12}(s) = \mp \frac{s^k}{g(s)}$$

$$e_{22}(s) = -(-1)^k \frac{h(-s)}{g(s)}.$$

5) Knowing $E = \{e_{ij}\}$, $(i,j) = 1,2$, compute the transducer power gain $T(\omega)$

$$T(\omega) = T_g \frac{|e_{21}|^2 |l_{21}|^2}{|1 - e_{11}S_G|^2 |1 - \hat{e}_{22}S_L|^2} \tag{5}$$

where

$$T_g = 1 - |S_G|^2 \qquad |l_{21}|^2 = 1 - |S_L|^2$$

$$\hat{e}_{22} = e_{22} + \frac{e_{21}^2 S_G}{1 - e_{11}S_G}.$$

A simple approach to optimize the transducer power gain $T(\omega)$ and thereby obtain the coefficients of $h(s)$ may be formulated by using the least-square method. The objective function δ may be written as

$$\delta = \sum_{i=1}^{m} \{T(\omega_i) - T_0\}^2 \tag{6}$$

where T_0 is the desired flat gain level to be approximated in the least-square sense, m is the number of sampling frequencies over the passband, and $T(\omega_i)$ is the transducer gain generated employing the previous algorithm in terms of the initialized coefficients h_{i0}. The error δ is minimized using a linear least-square minimization program (e.g., Levenberg–Marquard technique) to find correction increments Δh_i, then the initial coefficients are revised as $h_i = h_{i0} + \Delta h_i$ at each iteration. Once the final form of $e_{11}(s)$ is computed, it is realized by the Darlington procedure as a lossless 2-port ladder with resistive termination. The lossless 2-port is the equalizer E.

As is usually the case, an intelligent initial guess is important in efficiently running the program. In the present problem, the following initialization proved very successful. We presume, to start, that the problem under consideration is a single-matching problem; that is either source or load network is assumed to be purely resistive. Then, employing the real frequency technique for single-matching problems [1], the input impedance Z (or equivalently input admittance Y) of the lossless matching network with resistive termination is computed, and the corresponding reflection coefficient $e_{110}(s) = (Z-1)/(Z+1)$ is generated. The numerator of e_{110} is the initial choice for $h(s)$.

An ad hoc direct choice for coefficients h_i (e.g., $h_i = 1$ or -1) is, of course, not precluded in simpler problems.

It is important to note that there is no restriction other than reality imposed on the unknown coefficients (h_i, $i = 0 \cdots n$). Realizability is simply achieved as a consequence of spectral factorization yielding $g(s)$ as a Hurwitz polynomial.

III. Design of Multistage FET Amplifiers

Referring to Fig. 2(c) for the first k cascaded amplifier stages, the transducer power gain $T_k(\omega)$ is given by (see Appendix)

$$T_k(\omega) = T_{(k-1)}\left[\frac{|e_{21_k}|^2 |l_{21_k}|^2}{|1 - e_{11_k}S_{G_k}|^2 |1 - \hat{e}_{22_k}S_{l_k}|^2}\right]$$

$$= T_{(k-1)} \cdot E_K(\omega^2), \qquad k \geqslant 1 \tag{7}$$

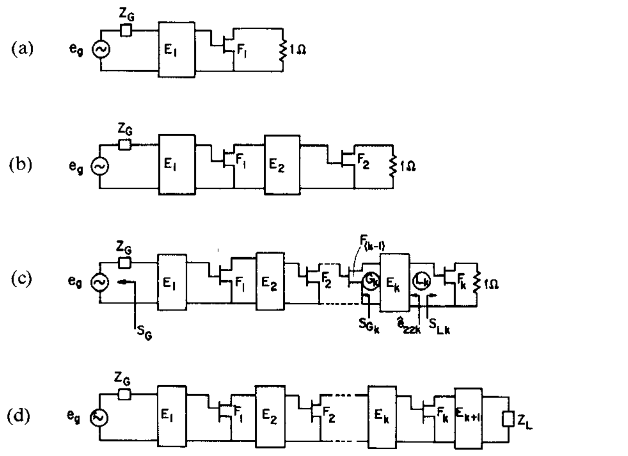


Fig. 2. Computation steps for designing broad-band multistage microwave FET amplifier. (a) Step 1: Design of front-end matching network E_1. (b) Step 2: Design of first interstage equalizer E_2. (c) Step k: Repeat Step 2 k times. (d) The last step: Design of back-end equalizer $E_{(k+1)}$.

where $T_{(k-1)}$ is the transducer power gain of the first $(k-1)$ stages with resistive terminations. $E_k(\omega)$ is the term in brackets [ ], e_{ij_k} are the unit normalized scattering parameters of the kth equalizer E_k, S_{G_k} is the unit normalized reflection coefficient measured at port G_K to the left, $\hat{e}_{22_k}$ and S_{l_k} are the unit normalized reflection coefficients measured at port L_K to the left and right, respectively. It is straightforward to generate $\hat{e}_{22_k}$ and S_{l_k} using the information obtained from the previous stages and the S parameter data for the FET's. Let $\{f_{ij_k}\}$ be the unit normalized scattering parameters of the kth FET (F_k). For the system shown in Fig. 2(c)

$$l_{21_k} = f_{21_k}, \qquad S_{l_k} = f_{11_k} \tag{8}$$

and

$$\hat{e}_{22_k} = e_{22_k} + \frac{e_{21_k}^2 S_{G_k}}{1 - S_{G_k} e_{11_k}} \tag{9}$$

where

$$S_{G_k} = f_{22_{(k-1)}} + \frac{f_{12_{(k-1)}} \cdot f_{21_{(k-1)}} \hat{e}_{22_{(k-1)}}}{1 - f_{11_{(k-1)}} \cdot \hat{e}_{22_{(k-1)}}}, \qquad k \geqslant 2 \tag{10}$$

with

$$S_{G_1} = \frac{Z_G - 1}{Z_{G+1}} \tag{11}$$

the known reflection coefficient of the source network. Utilizing the simplified real frequency technique described above, a multistage microwave FET amplifier may be designed as follows.

1) Construct the front-end equalizer $E_1 = \{e_{ij_1}\}$ for the first stage such that $T_1(\omega) = T_g \cdot E_1(\omega)$ is optimized (Fig. 2(a)), where $T_g = 1 - |S_{G_1}|^2$.

2) The second equalizer-FET stage is now cascaded with the first one. The interstage equalizer $E_2 = \{e_{ij_2}\}$ is computed so that $T_2(\omega^2) = T_1 \cdot E_2(\omega)$ is optimized. Notice that in $T_2(\omega)$, $T_1(\omega)$ is regarded as a weighting function (Fig. 2(b)).

3) Repeat Step 2 for designing the remaining cascaded stages (Fig. 2(c)) up to k.

4) Finally the back-end equalizer $E_{(k+1)} = \{e_{ij_{(k+1)}}\}$ is determined for a given complex load $S_L(j\omega)$ so that the overall transducer power gain $T(\omega)$

$$T(\omega) = (T_1 \cdot T_2 \cdots T_k) E_{(k+1)}(\omega) \tag{12}$$

is optimized. In (12) the term $(T_1 \cdot T_2 \cdots T_k)$ is known from the previous stages and acts as a weighting factor on $E_{(k+1)}(\omega)$ which is a function of the back-end equalizer (Fig. 2(d)), and given by replacing "k" with "$k+1$"$E_k(\omega)$ of (7). In this case

$$S_{l_{(k+1)}} = S_L = \frac{Z_L - 1}{Z_L + 1}, \qquad |l_{21_{(k+1)}}|^2 = 1 - |S_L|^2.$$

In the course of the above design process, the gain taper of each FET is compensated at the port connected to the preceding equalizer in the cascade. The term $E_{(k+1)}(\omega)$ in (12) only provides impedance matching. It should further be noted that (12) exactly specifies the gain with all equalizers and FET's in place, and the *nonunilateral behavior of the FET's taken into account*. However, the design procedure can be iterated one or more times in order to improve the maximum flat gain level.

The design technique that has been discussed is applicable to optimizing a variety of objective functions as in [2]. Thus it can be used for maximizing the minimum passband gain, or for minimizing maximum noise figure, or for noise measure design, etc.

Examples: In the present section we will exhibit an application of the above procedure. The following examples were solved using the computer program called CARMAN-03 which was developed at Cornell University by the authors and successfully adopted at Microwave Technology Center of RCA Laboratories—David Sarnoff Research Center with the support of Dr. H. Huang.

The example chosen here does not call for particularly wide-band specifications. However it should be recalled that any problem concerned with a finite bandwidth as distinguished from a single frequency falls within the province of gain bandwidth theory when the given physical system is loaded with reactive parasitics. In the present instance we used available data for a Mitsubishi, MGF 2124 FET (Table I) for illustrating a three-stage design over the band 11.7–12.2 GHz. The uncompensated and optimum single frequency performance of the FET gives some idea of the improvements effected by compensation in the final amplifier design. Thus a calculation from input data shows that uncompensated input VSWR of the device package hovers around 7.0 over the band and output VSWR around 3.3. The final three-stage design has an input VSWR which varies from about 1.7 to 3.0. This could have been still further reduced by using the input equalizer E for matching *only* rather than combining taper compensation and matching in the input stage design as mentioned in Item 1) of the design summary given above. The output VSWR of the final three-stage system ranges from about 1.8 to 2.2. The maximum unilateral gain of each FET is of the order of 3.6 dB as computed at each frequency in the band (unrealizable over the band by any fixed tuned network) so that the total maximum unilateral

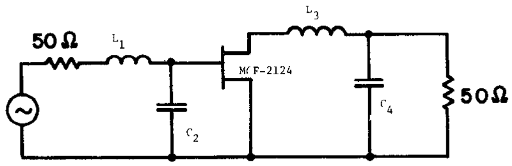


Fig. 3. Design of single-stage FET amplifier of Example 1. $L_1 = 1.23$ nH, $C_2 = 0.28$ pF, $L_3 = 0.146$ nH, $C_4 = 0.3$ pF.

TABLE I
MEASURED SCATTERING PARAMETERS (f_{ij}) OF MGF 2124

Freq. MHz	f_{11} MAG	f_{11} DEG	f_{21} MAG	f_{21} DEG	f_{12} MAG	f_{12} DEG	f_{22} MAG	f_{22} DEG
11700	.741	71.00	.865	- 91.8	.059	-74.8	.532	151.8
11800	.748	68.20	.883	- 94.8	.061	-76.5	.530	149.3
11900	.751	65.40	.886	- 97.4	.061	-79.6	.528	146.9
12000	.755	62.30	.895	- 99.6	.061	-82.5	.524	144.7
12100	.755	58.60	.910	-103.	.061	-84.7	.522	142.2
12200	.758	55.00	.905	-106.	.061	-86.1	.520	139.4

gain level is about 10.8 dB. The compensated three-stage system is 8.6 ± 1.0 over the band. Finally a glance at the f_{12} column of Table I shows that the FET device exhibits significant deviation from nonunilateral behavior. Aside from these practical considerations the example fully illustrates the real frequency design procedure.

The actual design then proceeds by first computing the equalizer for a single-stage, then a two-stage, and finally the three-stage FET amplifier as discussed in connection with (12). Throughout the examples $R_0 = 50\ \Omega$ was chosen as the normalization number and no ideal transformers are employed in the matching networks. Each equalizer is a low-pass ladder, i.e., $h(0) = 0$ for each of the $e_{11}(s)$ functions.

At stage k (Fig. 2(c)), the number designating flat gain level T_{0k}, which was to be approximated in the least squares sense, was estimated as

$$T_{0k} = \text{minimum of } \left\{ T_{k-1} \frac{|f_{21_k}|^2}{1 - |f_{11_k}|^2} \right\} \text{ over the passband.}$$

At the last step (Fig. 2(d)) the goal was to reach the flat gain level

$$T_0 = \text{minimum of } T_k \left\{ \frac{1}{1 - |f_{22_k}|^2} \right\} \text{ over the passband.}$$

In these problems the unknown coefficients h_i were simply initialized as $+1$ or -1 at each step of the design procedure.

Example 1. Design of single-stage FET amplifier:

Generator $S_{G_1} = 0\ (Z_G = 50\ \Omega)$
Load $S_L = 0\ (Z_L = 50\ \Omega)$
Passband $11.7\ \text{GHz} \leqslant f \leqslant 12.2\ \text{GHz}$ (X-Band)
Device Mitsubishi FET, MGF-2124 package. Scattering parameters $F = \{f_{ij}\}$, $(i, j) = 1,2$ are listed in Table I [6]. The complete design is depicted in Fig. 3.

Following the steps shown in Fig. 2, the front-end matching network E_1 is constructed when the FET is terminated with 50 Ω. $e_{11_1}(s)$ is found as

$$e_{11_1}(s) = \frac{0 + 0.404s + 1.0065s^2}{1 + 1.475s + 1.0065s^2}$$

and

$$T_1(\omega) \text{ is } 2.26 \pm 0.2 \text{ dB.}$$

The second and last step of this example is to design the back-end matching network E_2. The reflection coefficient $e_{11_2}(s)$ is optimized as

$$e_{11_2}(s) = \frac{0 - 0.461s + 0.129s^2}{1 + 0.686s + 0.129s}$$

and the overall gain $T(\omega)$ of the Fig. 3 system is 3.9 ± 0.21 dB.

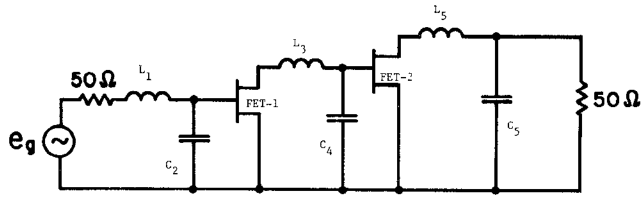


Fig. 4. Design of two-stage FET amplifier of Example 2. $L_1 = 1.23$ nH, $C_2 = 0.28$ pF, $L_3 = 0.485$ nH, $C_4 = 0.397$ pF, $L_5 = 0.253$ nH, $C_6 = 0.154$ pF.

Example 2. Design of two-stage FET amplifier:

Generator $S_{G_1} = 0\ (Z_G = 50\ \Omega)$
Load $S_L = 0\ (Z_L = 50\ \Omega)$
Passband $11.7\ \text{GHz} \leqslant f \leqslant 12.2\ \text{GHz}$ (X-Band)
Devices FET_1 and FET_2 are identical and given as in Table I.

The front-end equalizer E_1 has already been computed in Example 1. The second step is to realize the interstage matching network E_2 when E_1–FET_1 is cascaded with E_2–FET_2. Following the previous discussion and using the transducer gain (7) of the previous stage as a weighting function $e_{11_2}(s)$ is found as

$$e_{11_2}(s) = \frac{0 - 0.389s + 0.567s^2}{1 + 1.133s + 0.567s^2}.$$

At this step, $T_2(\omega)$ is 6.1 ± 0.6 dB. Finally, the back-end matching network is designed when E_1–FET_1–E_2–FET_2 is cascaded with E_3. e_{11_3} is computed as

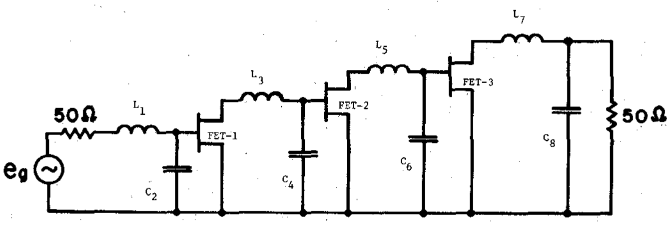
$$e_{11_3}(s) = \frac{0 - 0.1s + 0.1145s^2}{1 + 0.489s + 0.1145s^2}.$$

The overall two-stage amplifier is shown in Fig. 4 and the transducer power gain (TPG) is found as

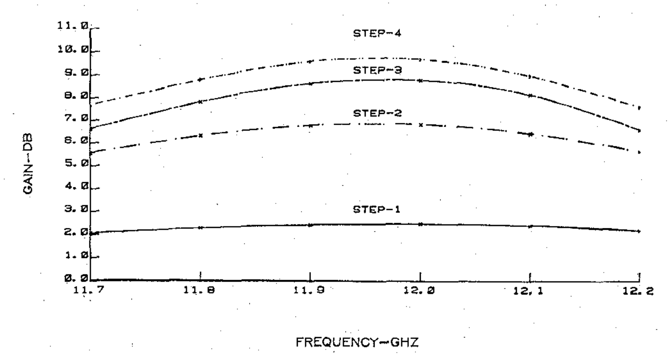
$$T(\omega) \cong 6.77 \pm 0.63 \text{ dB.}$$

Example 3. Three-stage FET amplifier design:

Generator $S_{G_1} = 0\ (Z_G = 50\ \Omega)$
Load $S_L = 0\ (Z_L = 50\ \Omega)$
Passband $11.7\ \text{GHz} \leqslant f \leqslant 12.2\ \text{GHz}$ (X-Band)
Devices FET_1, FET_2, FET_3 are identical and given as in Example 1 Table I.



(a)



(b)

Fig. 5. (a) Design of three-stage amplifier of Example 3. $L_1 = 1.23$ nH, $C_2 = 0.28$ pF, $L_3 = 0.485$ nH, $C_4 = 0.397$ pF, $L_5 = 0.093$ nH, $C_6 = 0.605$ pF, $L_7 = 0.286$ nH, $C_8 = 0.257$ pF. (b) Performance of the design steps of Example 3.

The first two steps of this example have been performed for Example 2 (E_1 and E_2 have been designed). In the third step, the interstage equalizer E_3 is constructed. Again using the gains of the previous stages as weighting functions

$$e_{11_3}(s) = \frac{0 - 1.088s + 0.166s^2}{1 + 1.232s + 0.166s^2}$$

and optimized gain function T_3 is 7.7 ± 1 dB.

Finally, the back-end equalizer E_4 is designed

$$e_{11_4}(s) = \frac{0 - 0.275s + 0.217s^2}{1 + 0.714s + 0.217s^2}.$$

The performance of the complete structure is $T(\omega) = 8.6 \pm 0.98$ dB (Fig. 5(b)) and the three-stage amplifier is shown in Fig. 5(a).

IV. Conclusion

The double-matching real frequency method described in this paper should be particularly useful in practical design problems. Loads and devices need only be specified by empirical data. The numerical procedures are arithmetically well behaved. Since a system transfer function class is not *a priori* used as in analytic theory, and further, since the method does not involve element values nor equalizer topology, the technique has wide flexibility in its range of application. Finally, even when analytic methods can apply, the real frequency technique generally yields simpler equalizers and superior operating characteristics.

Appendix

In this appendix, we show the derivation of the transducer power gain for cascaded, multistage FET amplifiers.

First assume that the generator is resistive. This restriction will later be removed without loss of generality.

Referring to Fig. 2(c), let $S_{(k-1)} = [s_{ij_{(k-1)}}]$, $E_k = [e_{ij_k}]$, and $F_k = [f_{ij_k}]$ be the scattering parameters of the $(k-1)$-cascaded stages, the kth equalizer, and the FET, respectively. Let us also denote the corresponding transmission matrices as $T_{k-1} = [t_{ij_{k-1}}]$, $T_{E_k} = [t_{ij_{E_k}}]$, and $T_{F_k} = [t_{ij_F}k]$, respectively. All normalization numbers are the same and real.

For any k-cascaded stages, the transmission matrix $T_k = [t_{ij_k}]$ is given as

$$T_k = \hat{T} \cdot T_{F_k} \tag{A1}$$

where the transmission matrix $\hat{T}$ is

$$\hat{T} = T_{(k-1)} \cdot T_{E_k} = [\hat{t}_{ij}]. \tag{A2}$$

The relation between the scattering and the transmission matrices is given in [7].

Let $T = [t_{ij}]$ be the transmission matrix of a 2-port and $S = [s_{ij}]$ be its corresponding scattering matrix. Then, the entries of matrix T are given as follows:

$$t_{11} = s_{12} - \frac{s_{11}s_{22}}{s_{21}} \qquad t_{12} = \frac{s_{11}}{s_{21}}$$

$$t_{21} = -\frac{s_{22}}{s_{21}} \qquad t_{22} = \frac{1}{s_{21}}. \tag{A3}$$

It should be emphasized that the inverse of t_{22} is directly related to the TPG of the 2-port ($|s_{21}|^2 = 1/|t_{22}|^2$).

Thus, employing (A3) and evaluating (A2) and (A1) sequentially, one calculates the transmission matrix of the k-cascaded stages.

The term t_{22} in (A2) is found as

$$t_{22} = \frac{1 - s_{22_{(k-1)}} \cdot e_{11_k}}{s_{21_{(k-1)}} \cdot e_{21_k}} = \frac{1}{\hat{s}_{21}} \tag{A4a}$$

or

$$\hat{s}_{21} = s_{21_{(k-1)}} \cdot \frac{e_{21_k}}{1 - s_{22_{(k-1)}} e_{11_k}} \tag{A4b}$$

where $\hat{s}_{21}$ is the transfer scattering parameter of the cascaded connection of $(k-1)$ stages with the kth matching network E_k.

Similarly, s_{21_k} is evaluated

$$s_{21_k} = \hat{s}_{21} \frac{F_{21_k}}{1 - \hat{s}_{22} F_{11_k}}. \tag{A5}$$

Employing (A4b) in (A5), one obtains the expression for the TPG of the k-cascaded stages

$$T_k = |s_{21_k}(j\omega)|^2 = |s_{21_{k-1}}|^2 \frac{|e_{21_k}|^2 |F_{21_k}|^2}{|1 - s_{22_{(k-1)}} e_{11}|^2 |1 - \hat{s}_{22_k} F_{11_k}|^2}. \tag{A6}$$

Note that throughout the text, the following notation has been used:

$$s_{22_{(k-1)}} = S_{G_{(k-1)}}$$
$$\hat{s}_{22_k} = \hat{e}_{22_k}$$
$$|s_{21_{(k-1)}}|^2 = T_{(k-1)}$$
$$|s_{21_k}|^2 = T_k. \quad \text{(A7)}$$

Complex Generator and Load Case

Assuming the complex generator and load networks are passive one-ports, one can represent generator and load networks as lossless-Darlington two-ports with resistive termination. Then, lossless generator and load two-ports may also be included in the cascaded structure. This new structure is also considered as resistively terminated, including the complex generator and load. It should be noted that with this modification, the transducer power gain of the overall amplifier (including complex generator and load) remains unchanged [3].

Let

$$G = [g_{ij}] \text{ and}$$
$$L = [l_{ij_{(k+1)}}]$$

be the unit normalized scattering parameters associated with the lossless two-ports in the Darlington representation of the generator and load networks, respectively.

Employing (A6), at the first step of the design procedure, TPG T_1 is calculated

$$T_1 = |g_{21}|^2 \frac{|e_{21_1}|^2 |F_{21_1}|^2}{|1 - g_{22} e_{11_1}|^2 |1 - \hat{e}_{22_1} F_{11_1}|^2}$$

where $g_{22} = S_{G_1}$ is the generator reflection coefficient, and by losslessness

$$|g_{21}|^2 = 1 - |S_{G_1}|^2$$

which was defined as Tg.

Similarly, at the last step, TPG of the overall amplifier is given as

$$T = T_k \frac{|e_{21_{(k+1)}}|^2 |l_{21_{(k+1)}}|^2}{|1 - e_{11_{(k+1)}} S_{G_{(k+1)}}|^2 |1 - \hat{e}_{22_{k+1}} l_{11(k+1)}|^2}$$

where $[e_{ij_{(k+1)}}]$ is the scattering parameter of the back-end matching network, $l_{11(k+1)} = S_L$ is the reflection coefficient of the load, and by losslessness

$$|l_{21(k+1)}|^2 = 1 - |S_L|^2.$$

$e_{22(k+1)}$ and $S_{G_{(k+1)}}$ are then evaluated using (9) and (10).

Thus, we have completed the derivations of the transducer power gains for all the $(k+1)$ steps of the design procedure as given in Section III.

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